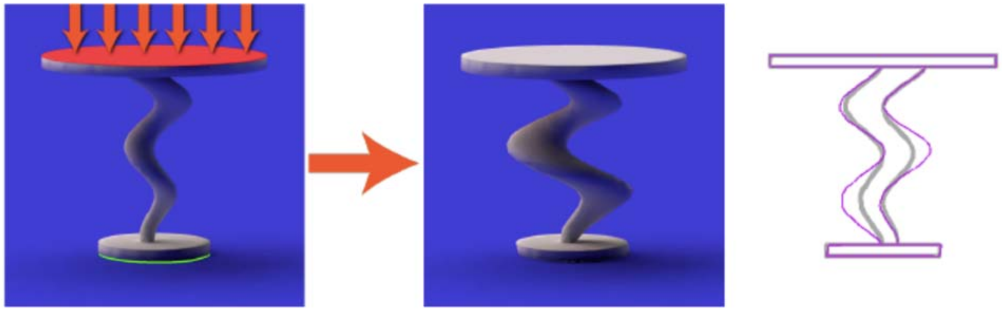
UMassAmherst

Direct Shape Optimization for Strengthening 3D Printable Objects



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University of Massachusetts Amherst

Introduction

- Printable 3D objects that need to withstand significant external force.

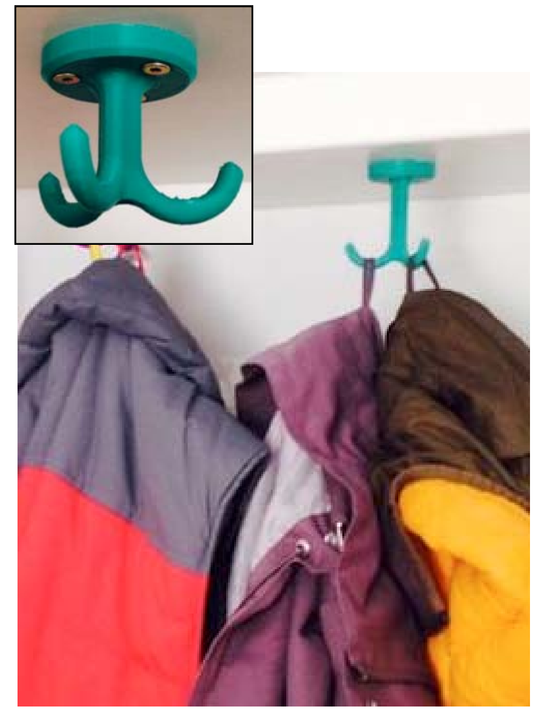
Coffee Table

Wrench

Coat Hanger

Introduction

- Printable 3D objects that need to withstand significant external force.

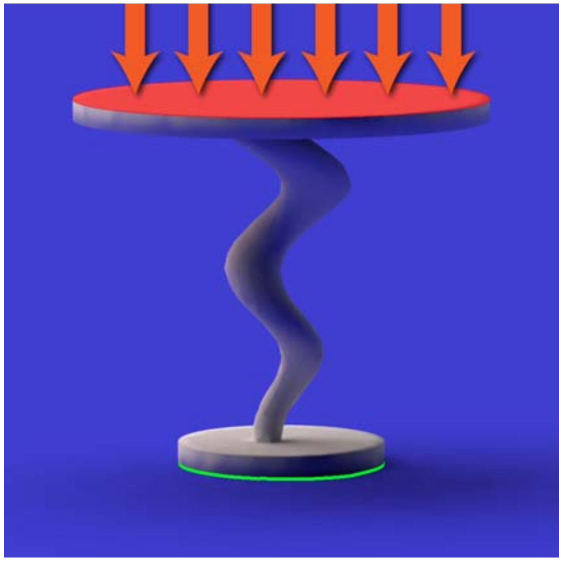


Introduction

- Given an input 3D shape,

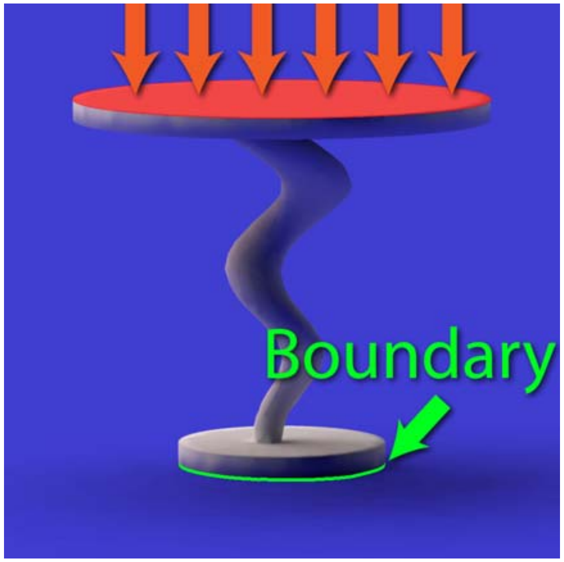
Introduction

- Given an input 3D shape, the direction and strength of external force,



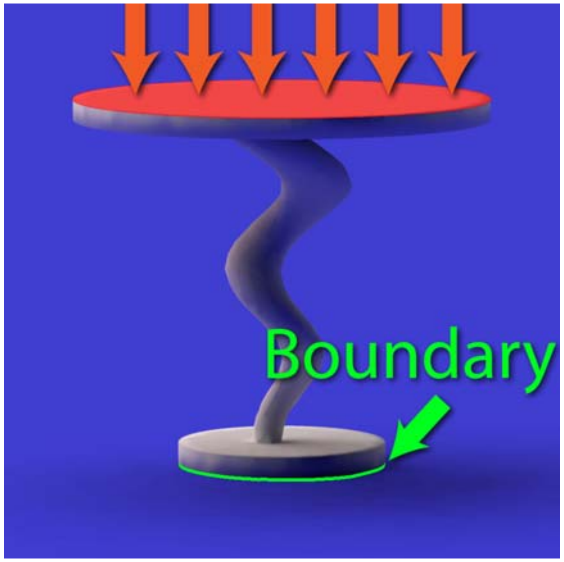
Introduction

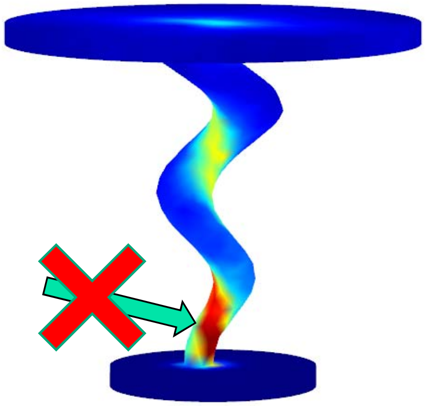
- Given an input 3D shape, the direction and strength of external force, and the boundary,



Introduction

- The object will deform under such external force. The regions under high stress may break.





Stress Analysis

Introduction

- Our goal is to optimize the shape, such that the resulting shape can successfully withstand the external force, and remain as similar as possible to the input shape.

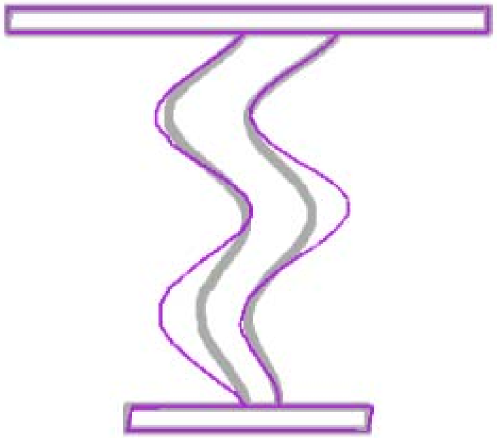
Optimized

Stress Analysis

Introduction

- Silhouette image showing the difference between the original (gray) and optimized (purple) shape.

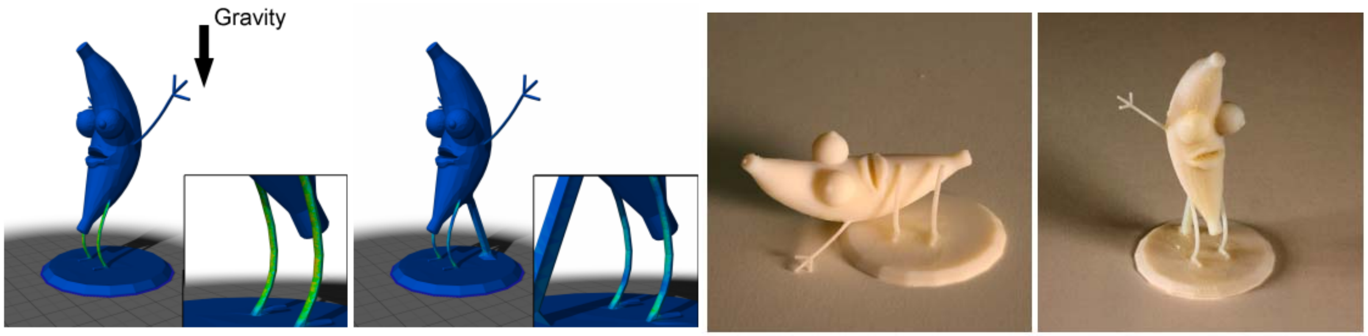
Optimized



Original vs. Optimized

Related Work

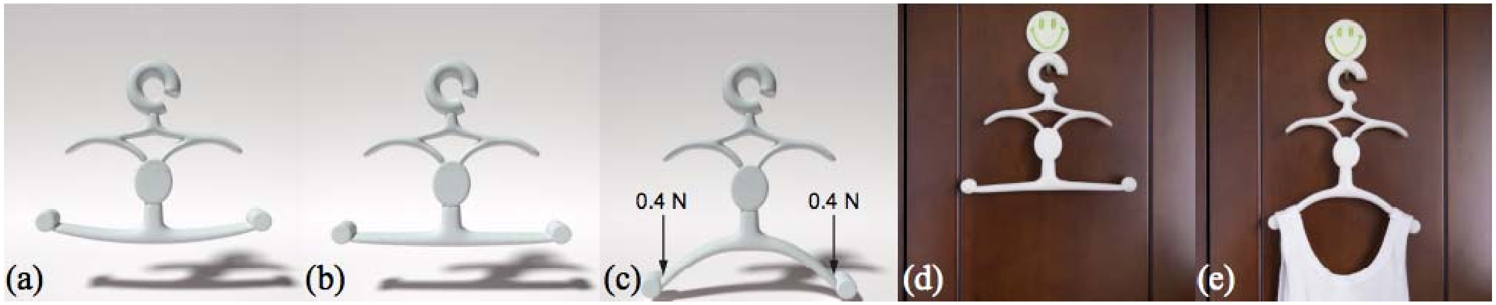
- Strengthening Printable Objects
  - **Hollowing** [SVB*12, LSZ*14, VGB*14]
  - **Internal skin framing** [WWY*13]
  - **Support struts** [SVB*12]
  - **Change printing directions** [HBA13, US13]
  - **Part thickening** [SVB*12, US13]
  - **Controllable shape design** [MDLW15]



[SVB*12]

Related Work

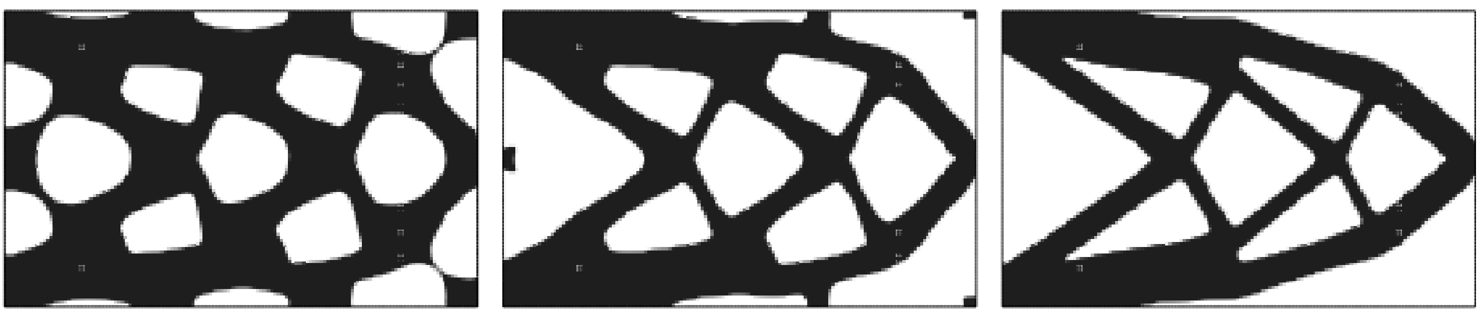
- Shape Optimization in Computer Graphics
  - **Balance** [PWLSH13]
  - **Spinnability** [BWBSH14]
  - **Aggregate mass** [MAB*15]
  - **Inverse elastic shape design** [CZXZ14]
  - **Microstructures** [PZM*15, SBR*15]



[CZXZ14]

Related Work

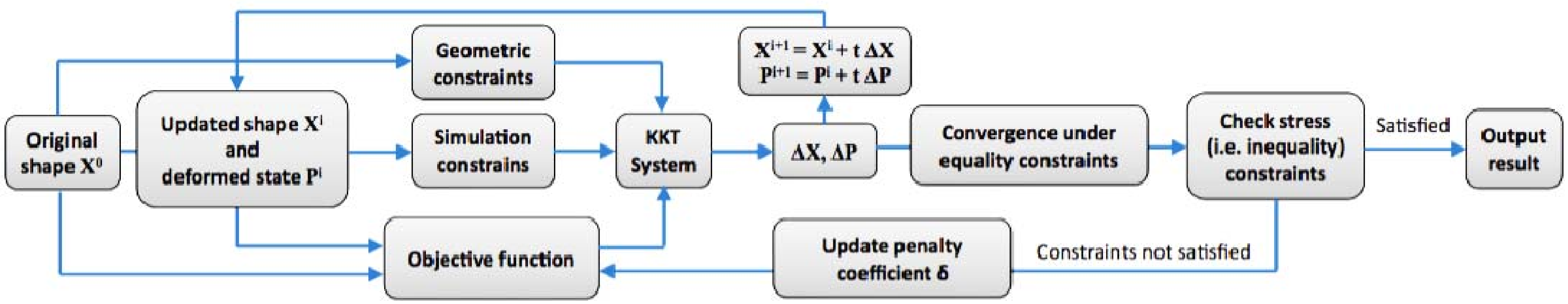
- Shape Optimization in Mechanical Engineering
  - **Parametric surface** [HM03, WMC08, BCC*10]
  - **B-splines/Bezier, subdivision surfaces** [BRC16]
  - **Level-set** [AJT02, AJ08, DMLK13]
  - **Specialized topology modifications** [All12, BS13]
  - **Procedural models** [BR88, RG92, Tor93]



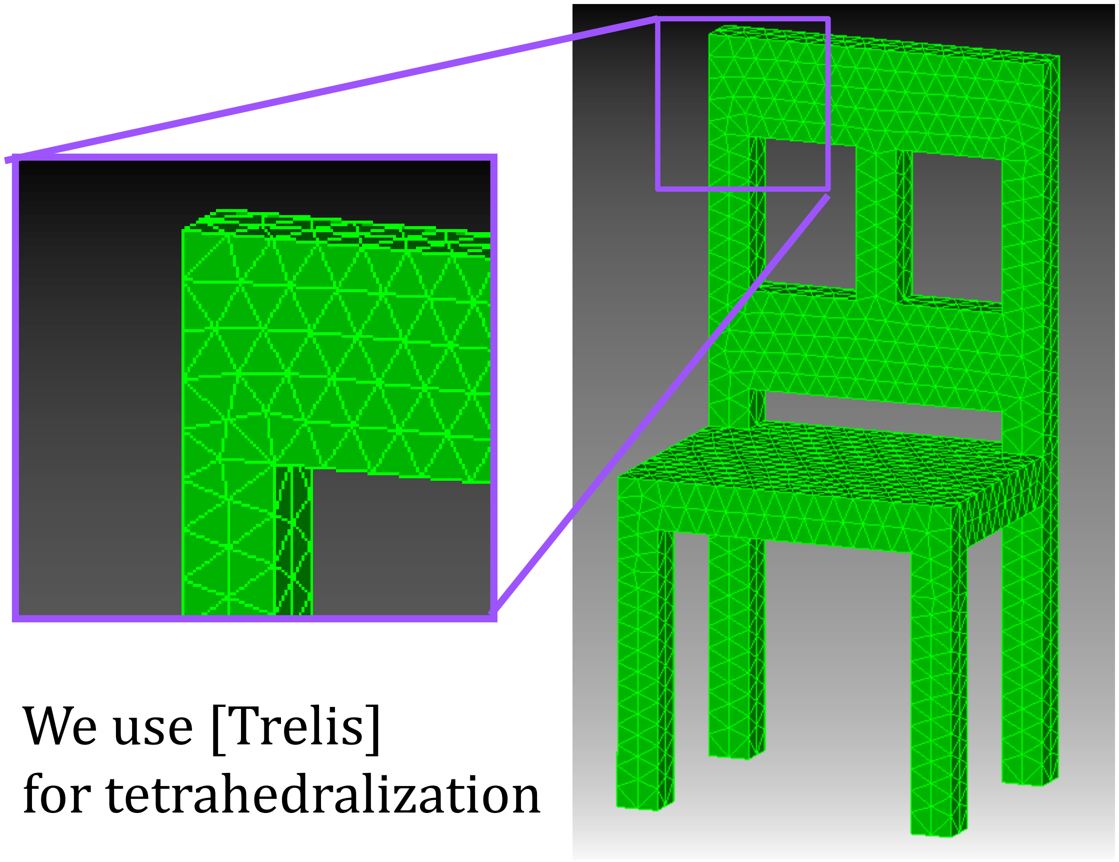
[All12]

Contributions

- A general, extensible method to optimize 3D shapes under physical and geometric constraints.
- Operates directly on the input mesh.
- Integrated physics simulation with optimizer.
  - Derivations of analytic gradient and Hessian



Overview



We use [Trelis]
for tetrahedralization

Overview

$\boldsymbol{X}_0$ Reference State | $\boldsymbol{X}$ Rest State | $\boldsymbol{P}$ Deformed State

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Overview

$\boldsymbol{X}_0$ Reference State

$\boldsymbol{X}$ Rest State

$\boldsymbol{P}$ Deformed State

Overview

$\boldsymbol{X}_0$
Reference State

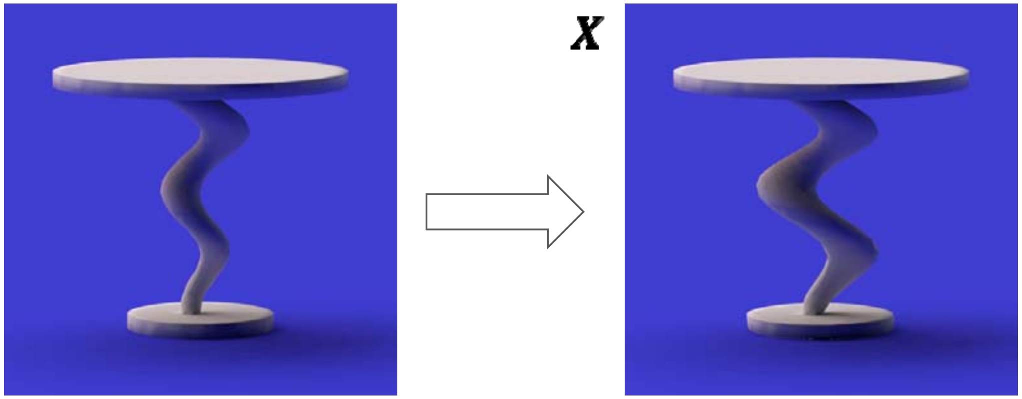
$\boldsymbol{X}$
Rest State

$\boldsymbol{P}$
Deformed State

Formulation – Constrained Optimization

Solve for rest state *X* that minimize an objective function while satisfying the given constraints.

X_0



X

Objective: $argmin_X \, D(\boldsymbol{X}, \boldsymbol{X}^0)$

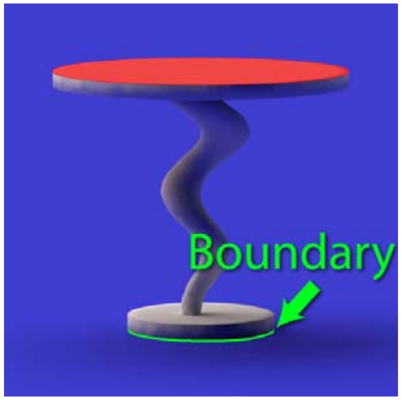
Constraints: $\forall v \in B: \boldsymbol{x}_v = \boldsymbol{p}_v, \forall v \notin B: \boldsymbol{f}_v(\boldsymbol{X}, \boldsymbol{P}) = \boldsymbol{0}$ **Simulation**

$\forall t: \hat{\sigma}_t^2(\boldsymbol{X}, \boldsymbol{P}) < C$ **Stress**

$\mathrm{g}(\mathbf{X}, \mathbf{X}^0) = 0$ **Geometric**

1. Simulation Constraints

$\boldsymbol{X}_0$
Reference State

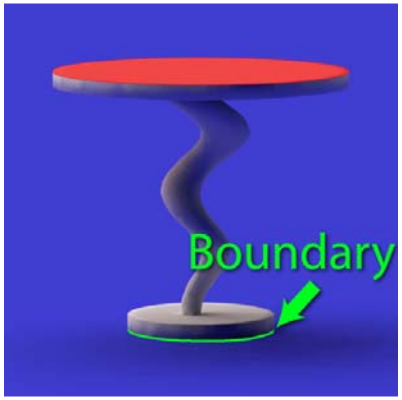


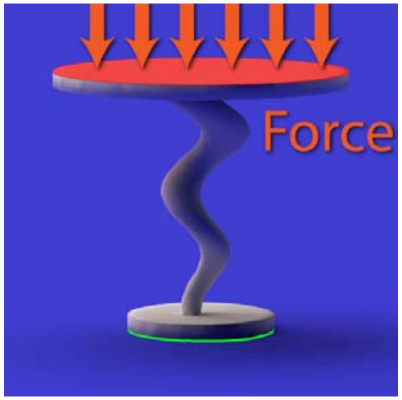
$$\forall v \in B: \boldsymbol{x}_v = \boldsymbol{p}_v, \forall v \notin B: \boldsymbol{f}_v(\boldsymbol{X}, \boldsymbol{P}) = \boldsymbol{0}$$

Boundary Conditions

1. Simulation Constraints

$\boldsymbol{X}_0$
Reference State



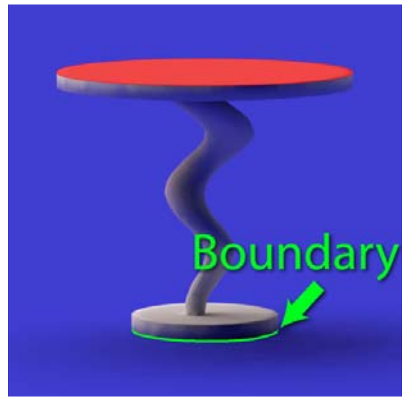


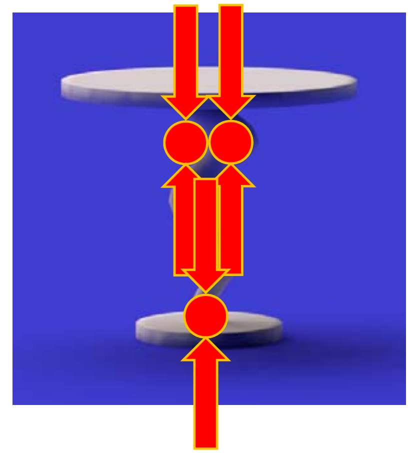
$$\forall v \in B: \boldsymbol{x}_v = \boldsymbol{p}_v, \forall v \notin B: \boldsymbol{f}_v(\boldsymbol{X}, \boldsymbol{P}) = \boldsymbol{0}$$

Force Equilibrium

1. Simulation Constraints

$\boldsymbol{X}_0$
Reference State

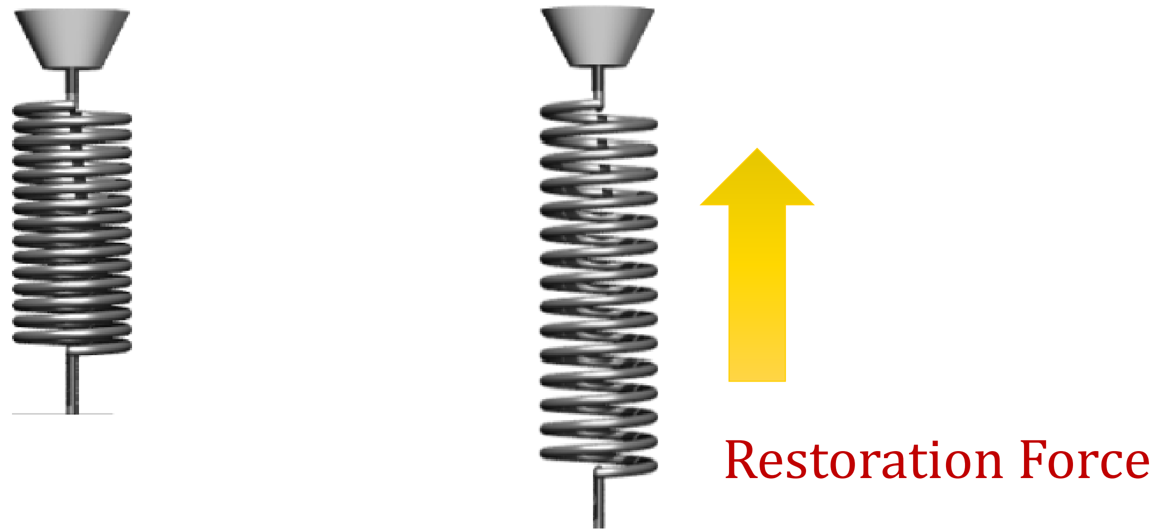




$$\forall v \in B: \boldsymbol{x}_v = \boldsymbol{p}_v, \underline{\forall v \notin B: \boldsymbol{f}_v(\boldsymbol{X}, \boldsymbol{P}) = \boldsymbol{0}}$$

Force Equilibrium

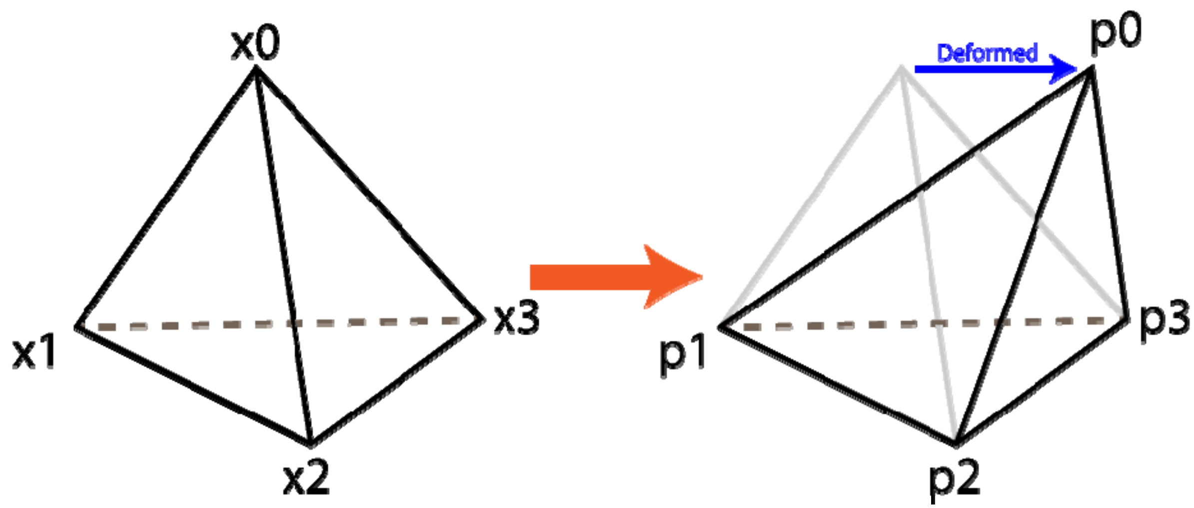
Elasticity Model



X
Rest State

P
Deformed State

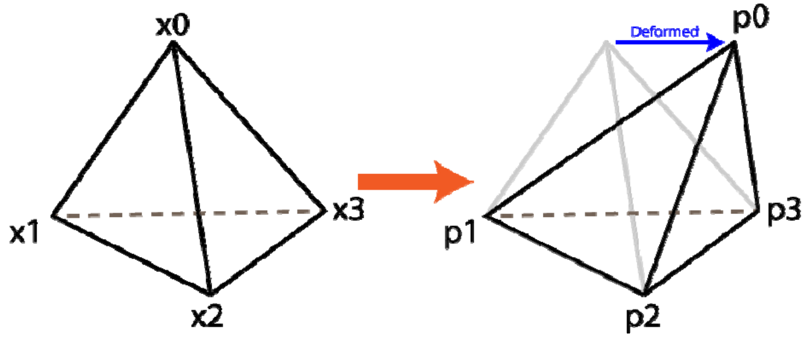
Elasticity Model



X
Rest State

P
Deformed State

Elasticity Model



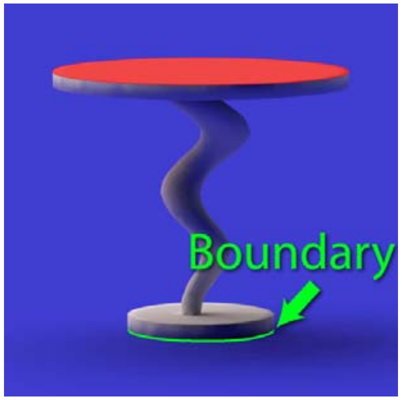
$$f_{t,v}(\boldsymbol{X}, \boldsymbol{P}) = \frac{\partial U_t(\boldsymbol{X}, \boldsymbol{P})}{\partial \boldsymbol{p}_v}$$

Strain Energy: $$U_t(\boldsymbol{X}, \boldsymbol{P}) = \frac{V_t}{2} \boldsymbol{\varepsilon}_t : \boldsymbol{E}\, \boldsymbol{\varepsilon}_t$$

Strain tensor Material tensor

1. Simulation Constraints

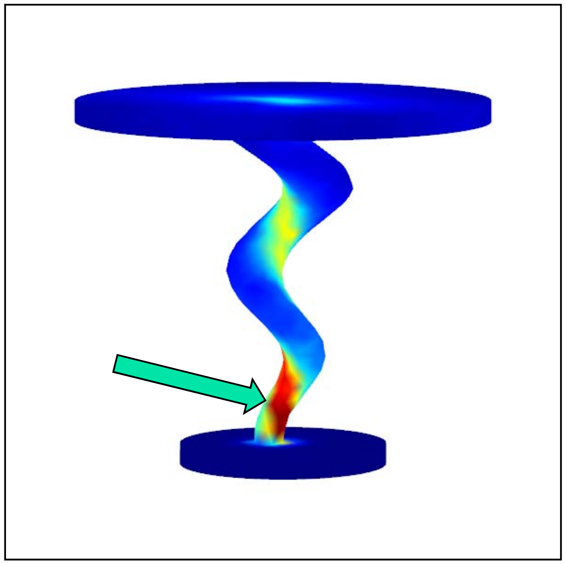
$\boldsymbol{X}_0$
Reference State



$$\underbrace{\forall v \in B: \boldsymbol{x}_v = \boldsymbol{p}_v}_{\text{Boundary Conditions}}, \underbrace{\forall v \notin B: \boldsymbol{f}_v(\boldsymbol{X}, \boldsymbol{P}) = \boldsymbol{0}}_{\text{Force Equilibrium}}$$

Boundary Conditions **Force Equilibrium**

2. Stress Constraints



$$\forall t : \hat{\sigma}_t^2(\boldsymbol{X}, \boldsymbol{P}) < C$$

von Mises Stress **Material's Yield Strength**

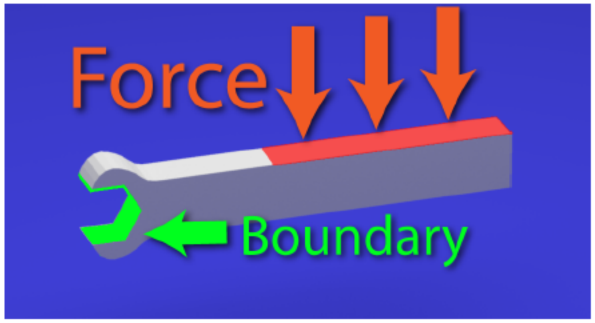
3. Geometric Constraints

- **Symmetry** Constraints: $\boldsymbol{S}_m \boldsymbol{x}_i = \boldsymbol{x}_j$
- **Interior Uniformity** Constraints: $\boldsymbol{x}_i = \frac{1}{|\mathbb{N}(i)|} \sum_{j \in \mathbb{N}(i)} \boldsymbol{x}_j$
- **User-Defined** Constraints

- All in the form of linear, equality constraints:

$$\mathrm{g}(\mathbf{X}, \mathbf{X}^0) = 0$$

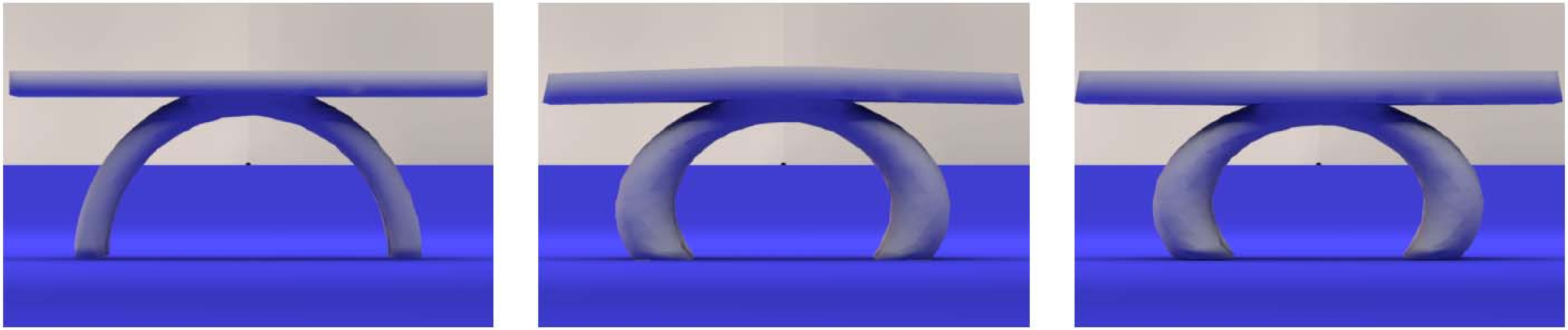
Symmetry Constraints



(a) original shape

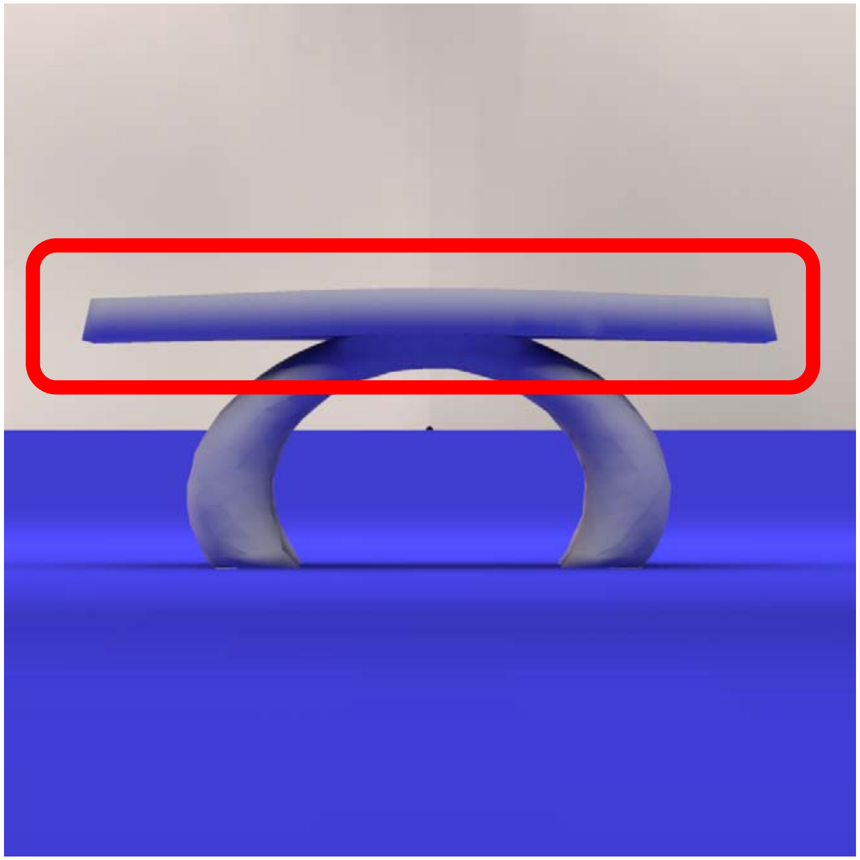
$$\mathbf{S}_m \mathbf{x}_i = \mathbf{x}_j$$

User-Defined Constraints

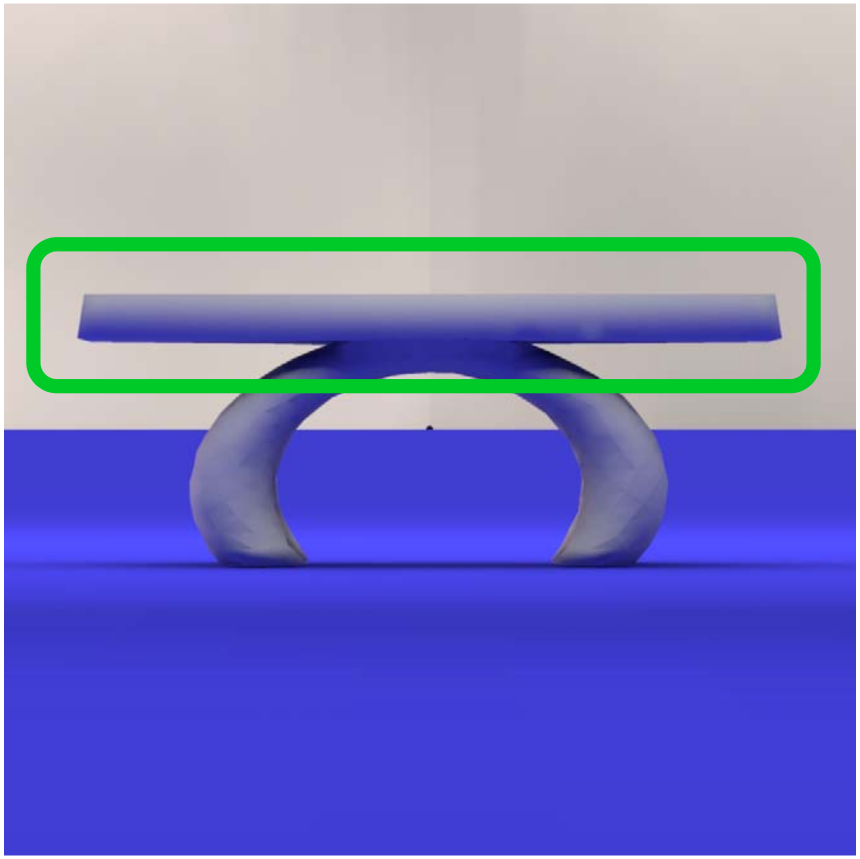


(a) original shape (b) w/o user constraints (c) with user constraints

Without User-Defined Constraints



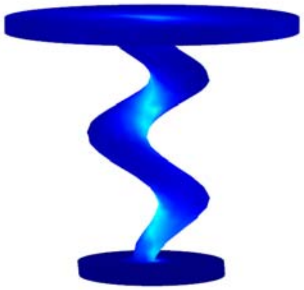
With User-Defined Constraints



Objective Function

X_0

X



P

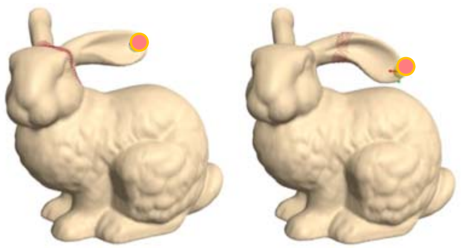
Constraints: $\forall v \in B: x_v = p_v, \forall v \notin B: f_v(X, P) = 0$ Simulation
$\forall t: \hat{\sigma}_t^2(X, P) < C$ Stress
$g(X, X^0) = 0$ Geometric

Objective Function

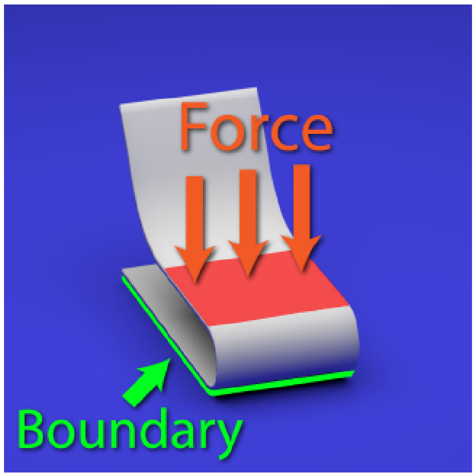
Objective: $argmin_{\boldsymbol{X}}\, D(\boldsymbol{X}, \boldsymbol{X}^0)$

$$D(\mathbf{X}, \mathbf{X}^{\mathbf{0}}) = w_1\, D_{intrinsic}(\mathbf{X}, \mathbf{X}^0) + w_2\, D_{extrinsic}(\mathbf{X}, \mathbf{X}^0)$$

- **Extrinsic**: L2 distance between vertices
  - Preserves overall shape
- **Intrinsic**: transformed surface Laplacians
  - Preserves surface details and smoothness
  - [Sorkine et al. 2004]



Objective Function



(a) original shape

Solving the Optimization

Objective: $argmin_{\boldsymbol{X}}\, D(\boldsymbol{X}, \boldsymbol{X}^0)$

Constraints: $\forall v \in B{:}\, \boldsymbol{x}_v = \boldsymbol{p}_v, \forall v \notin B{:}\, \boldsymbol{f}_v(\boldsymbol{X}, \boldsymbol{P}) = \boldsymbol{0}$ **Simulation**

→ $\forall t{:}\, \hat{\sigma}_t^2(\boldsymbol{X}, \boldsymbol{P}) < C$ **Stress**

$g(\mathbf{X}, \mathbf{X}^0) = 0$ **Geometric**

- **Stress Constraints are Inequality constraints**:
  - Use the penalty method.

The Penalty Method

$h(\cdot) = \min(0, \cdot)^2$

Objective: $argmin_{\boldsymbol{X}}\, D(\boldsymbol{X}, \boldsymbol{X}^0) + \delta \cdot \sum_t h(C - \hat{\sigma}_t^2(\boldsymbol{X}, \boldsymbol{P}))$

Constraints: $\forall v \in B: \boldsymbol{x}_v = \boldsymbol{p}_v, \forall v \notin B: \boldsymbol{f}_v(\boldsymbol{X}, \boldsymbol{P}) = \boldsymbol{0}$

$\mathbf{g}(\mathbf{X}, \mathbf{X}^0) = 0$

- h is a penalty function
- δ is the penalty weight
  - We start from a small weight and progressively increase it across iterations.

KKT System and Newton's Method

Objective: $argmin_{\boldsymbol{X}}\, D(\boldsymbol{X}, \boldsymbol{X}^0) + \delta \cdot \sum_t h(C - \hat{\sigma}_t^2(\boldsymbol{X}, \boldsymbol{P}))$

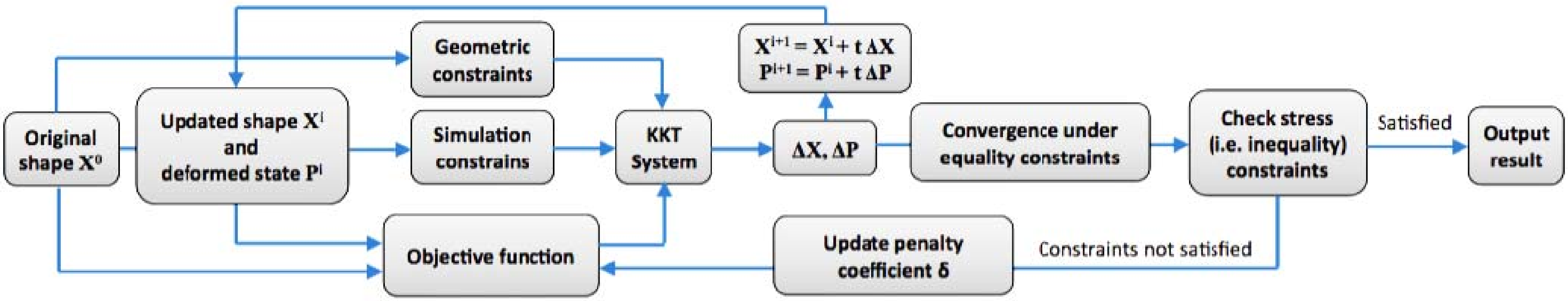
Constraints: $\forall v \in B: \boldsymbol{x}_v = \boldsymbol{p}_v, \forall v \notin B: \boldsymbol{f}_v(\boldsymbol{X}, \boldsymbol{P}) = \boldsymbol{0}$

$$\mathrm{g}(\mathbf{X}, \mathbf{X}^0) = 0$$

$$\begin{bmatrix} H & J^T \\ J & 0 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{x} \\ \mathbf{w} \end{bmatrix} = - \begin{bmatrix} \mathbf{g} \\ \mathbf{b} \end{bmatrix}$$

- $\mathbf{g}, H$: gradient and Hessian of the objective
- $\mathbf{b}, J$: function value and Jacobian of constraints

Algorithm Diagram

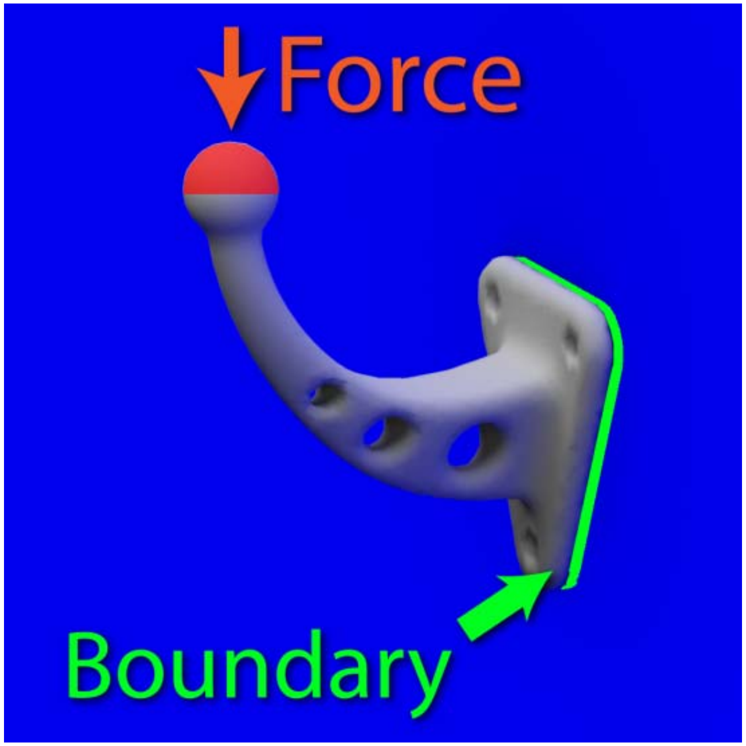


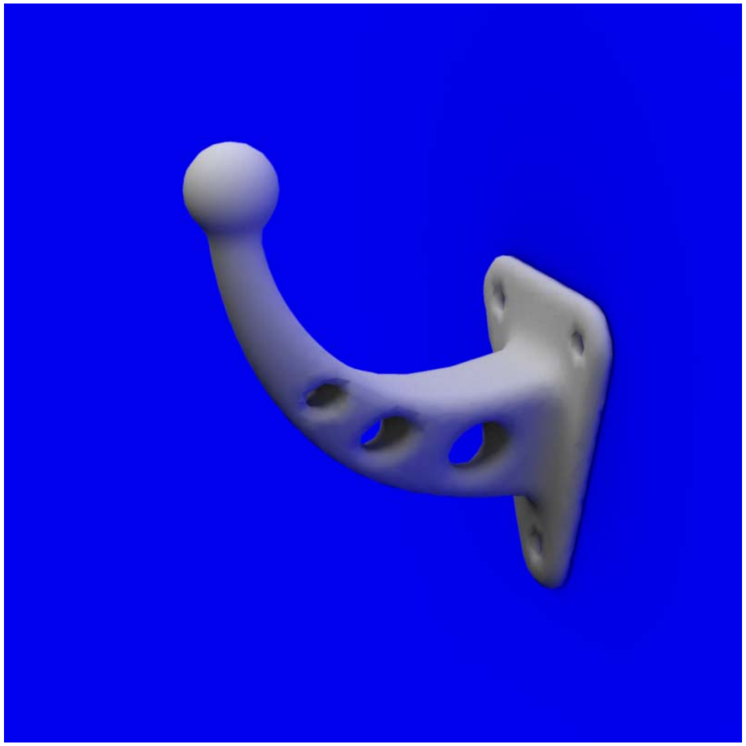
- Source code, data, and supplemental materials available for download from our paper website.

Results

- Coat Hanger Example:

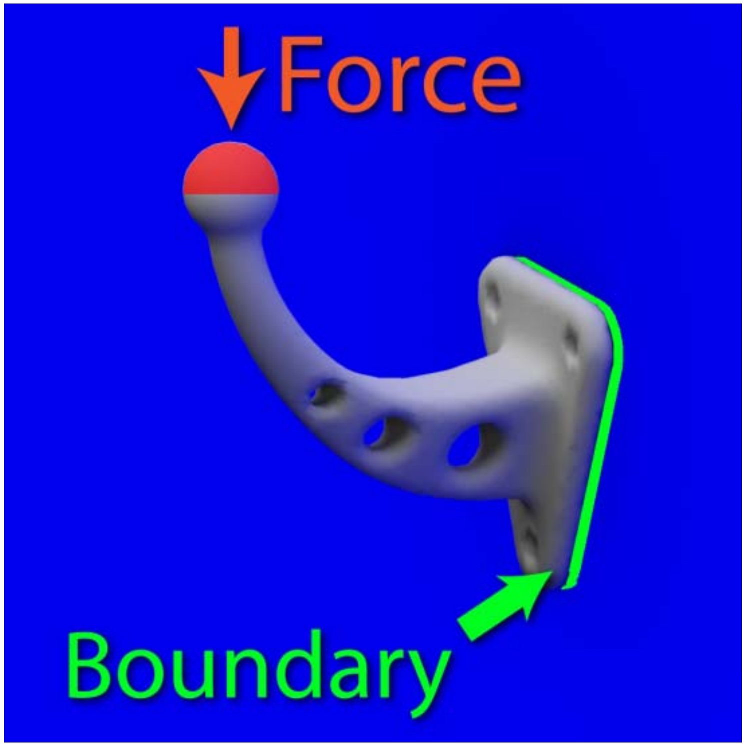
Endure 50% more force



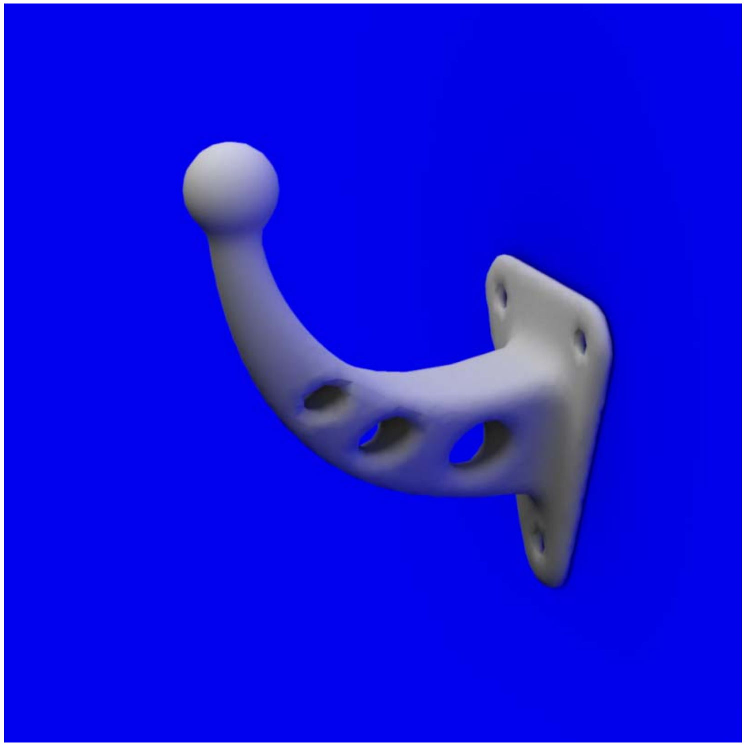


Results

- Coat Hanger Example:



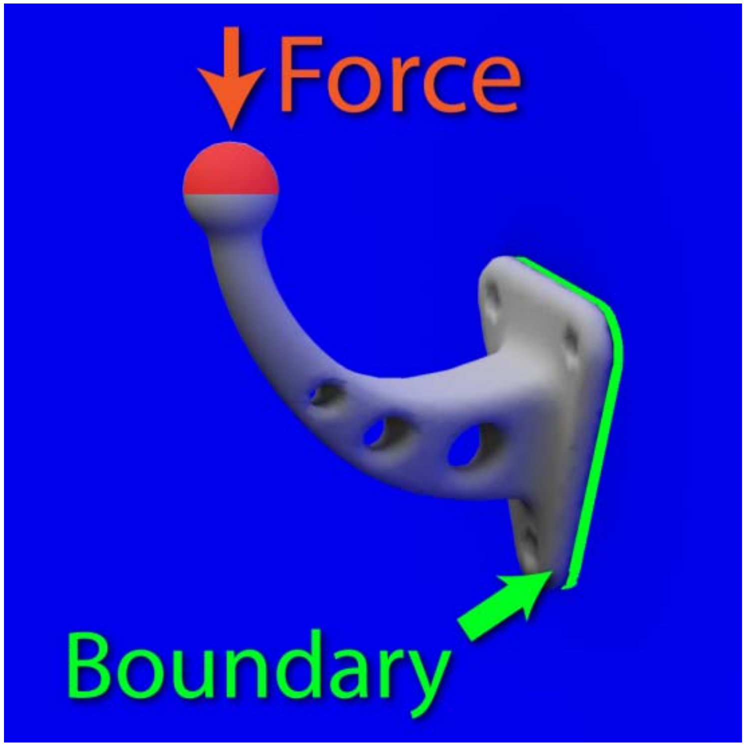
100% more

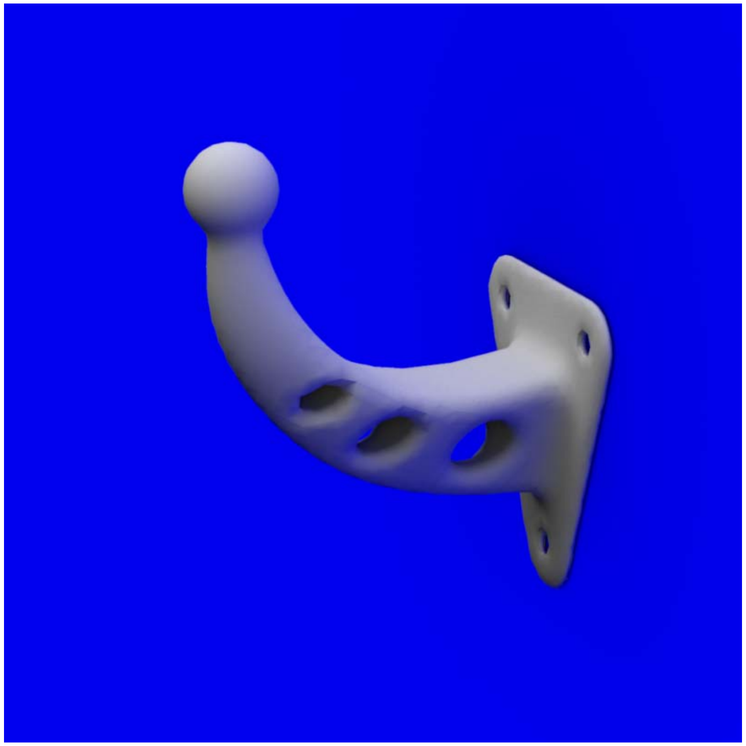


Results

- Coat Hanger Example:

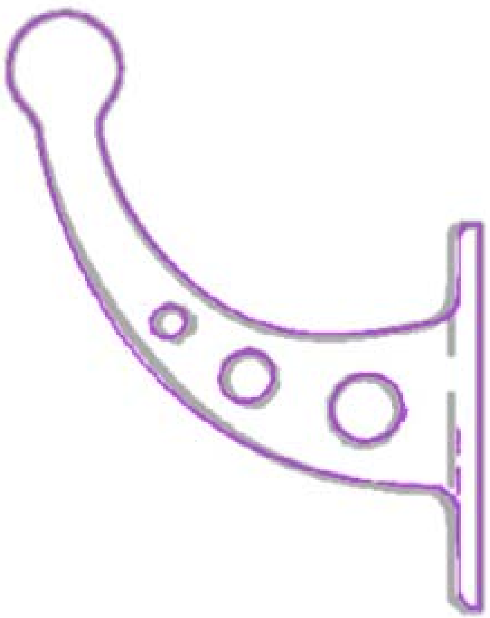
200% more



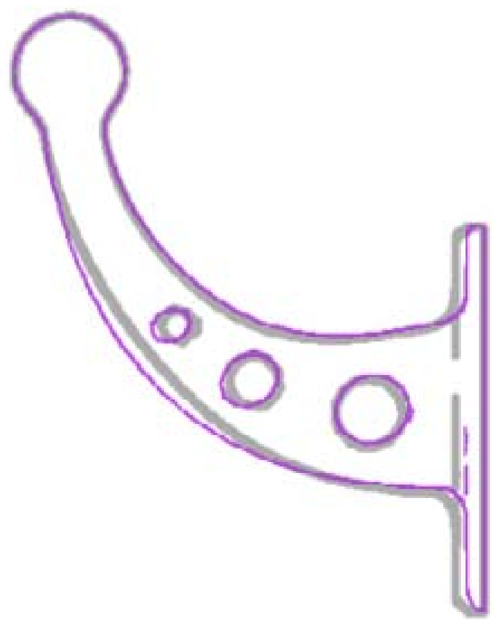


Results

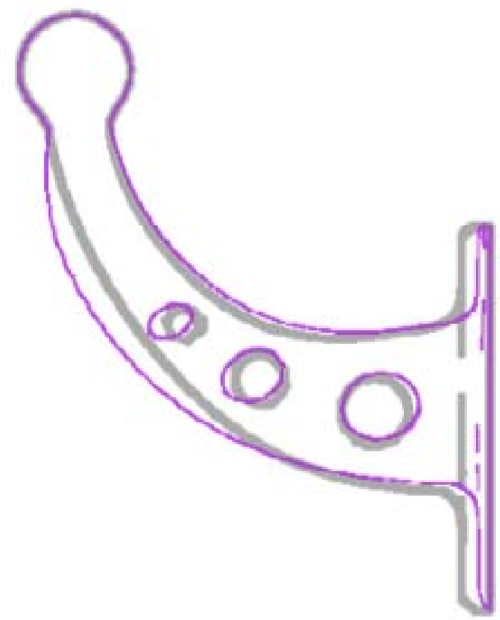
- Coat Hanger Example:



50% more

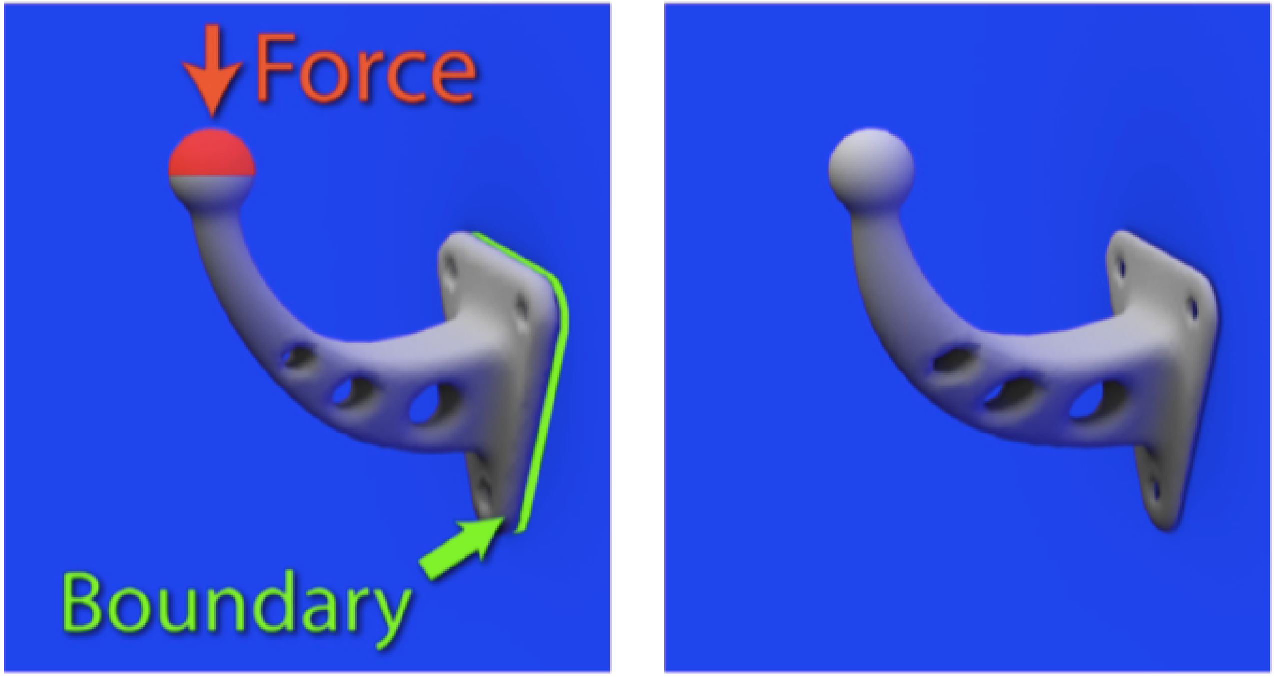


100% more



200% more

Comparison with Local Thickening

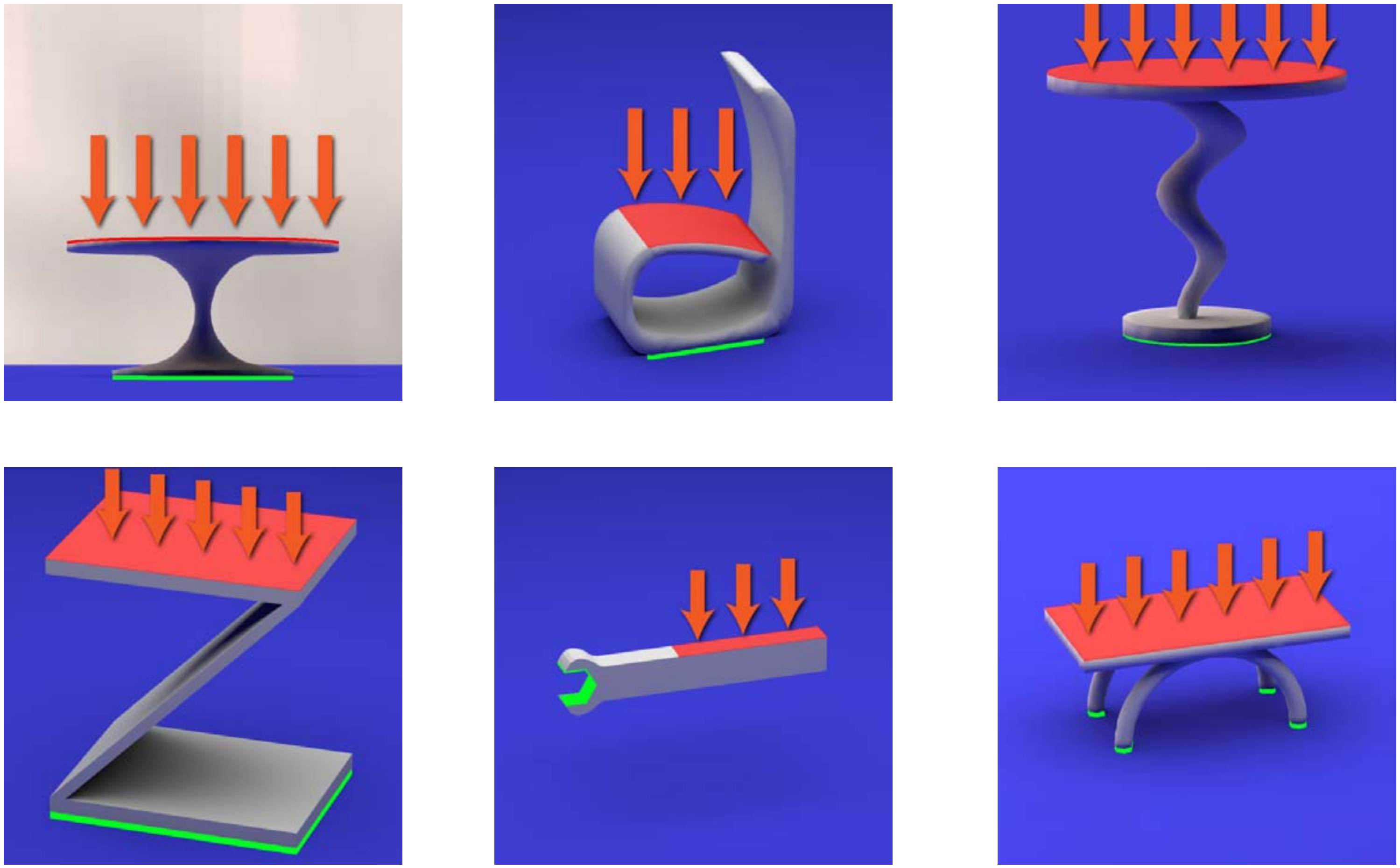


(a) Original shape

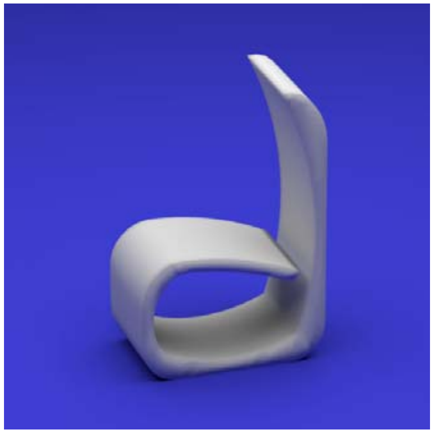
(b) Our method

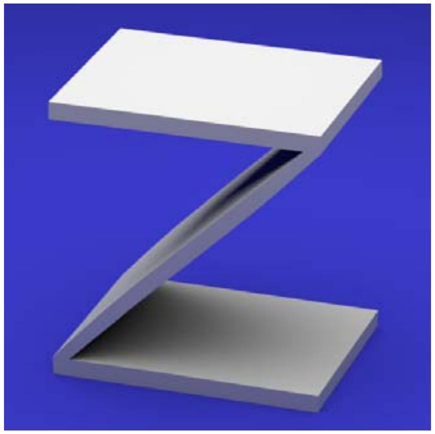
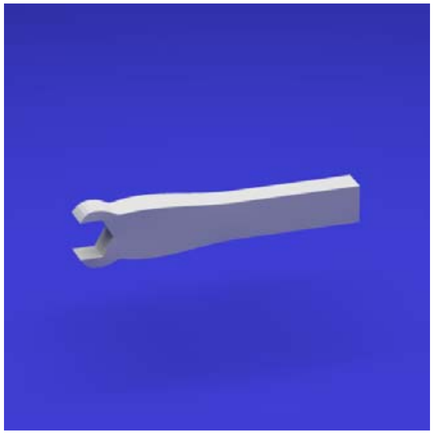
20% less material
Better at preserving surface features

Gallery of Results – Force and Boundary

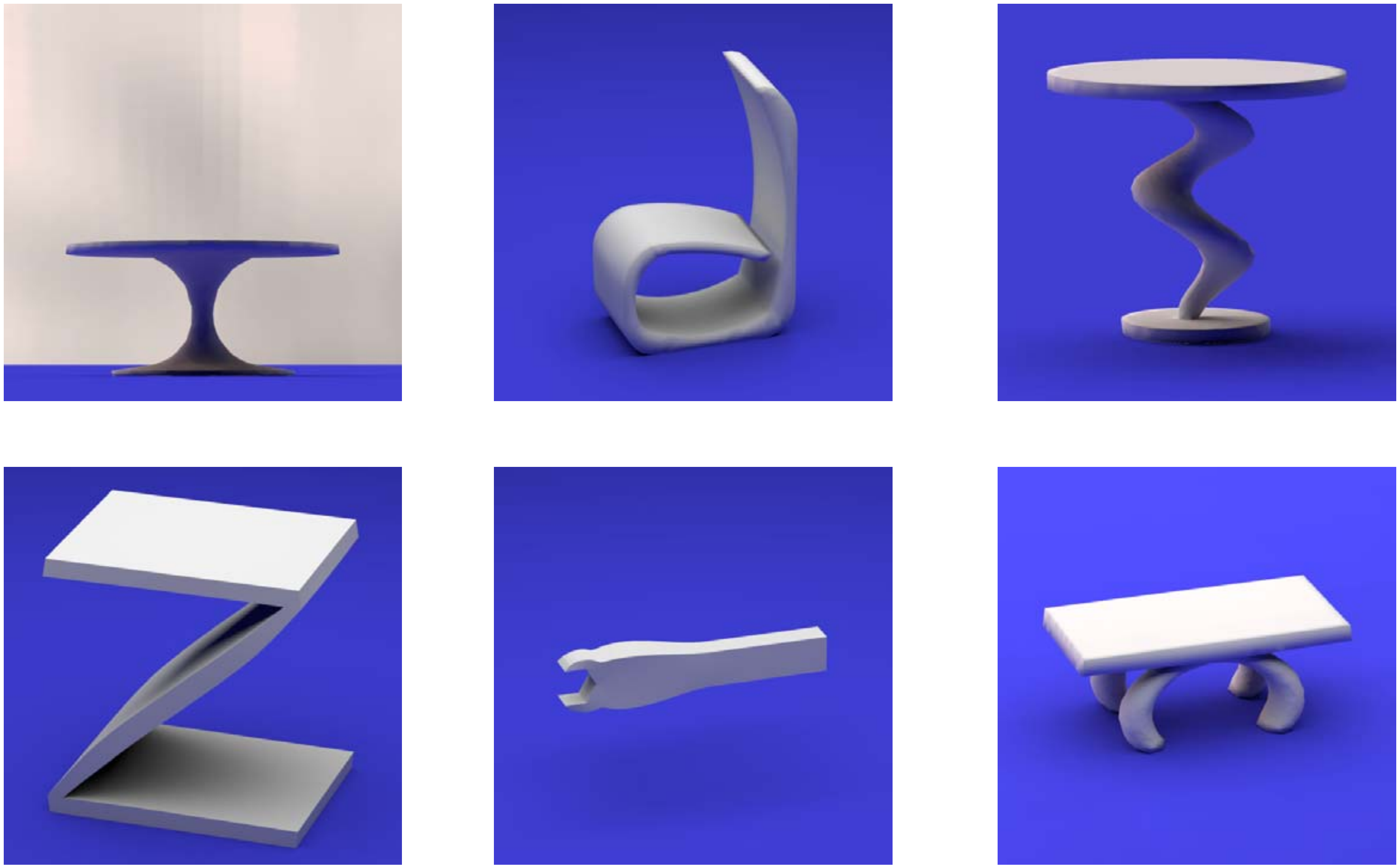


Gallery of Results – 50% More Force

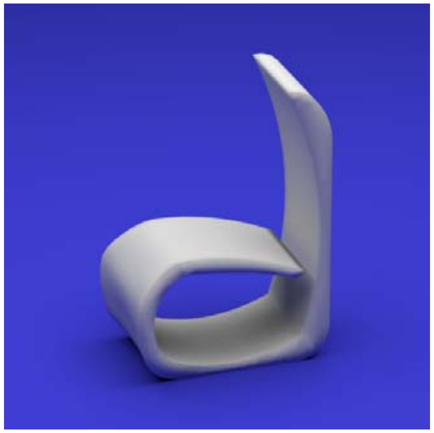



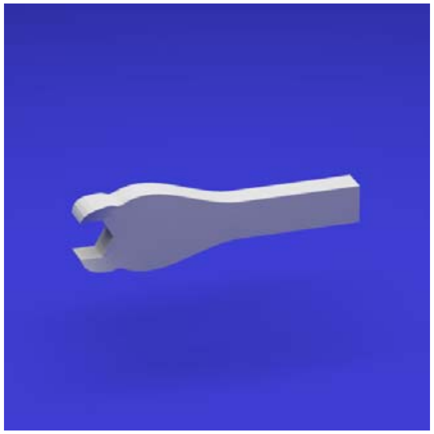




Gallery of Results – 100% More Force

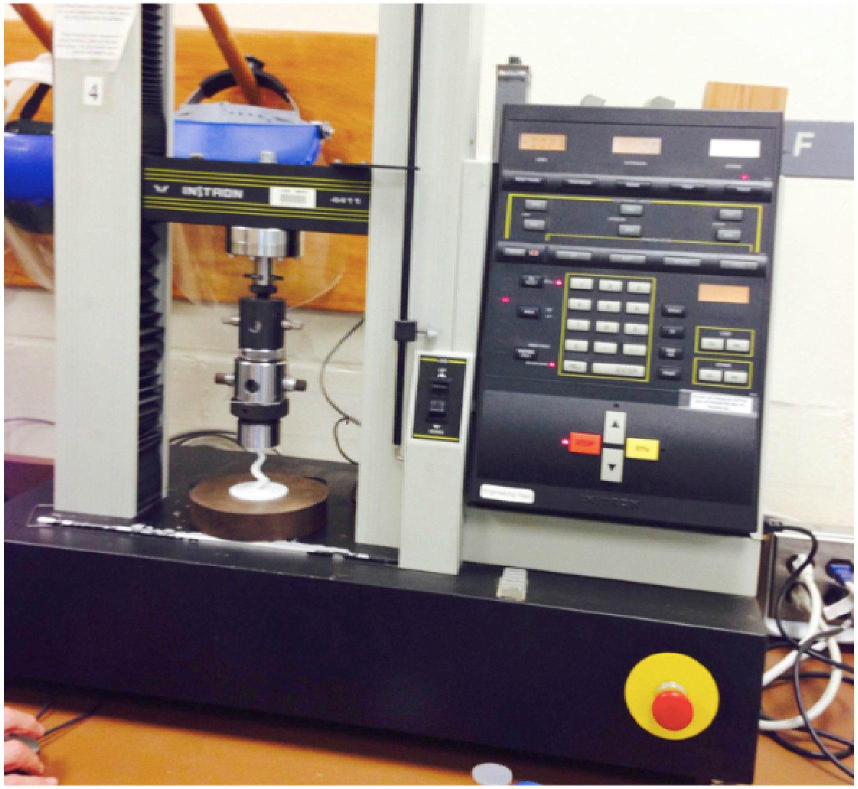


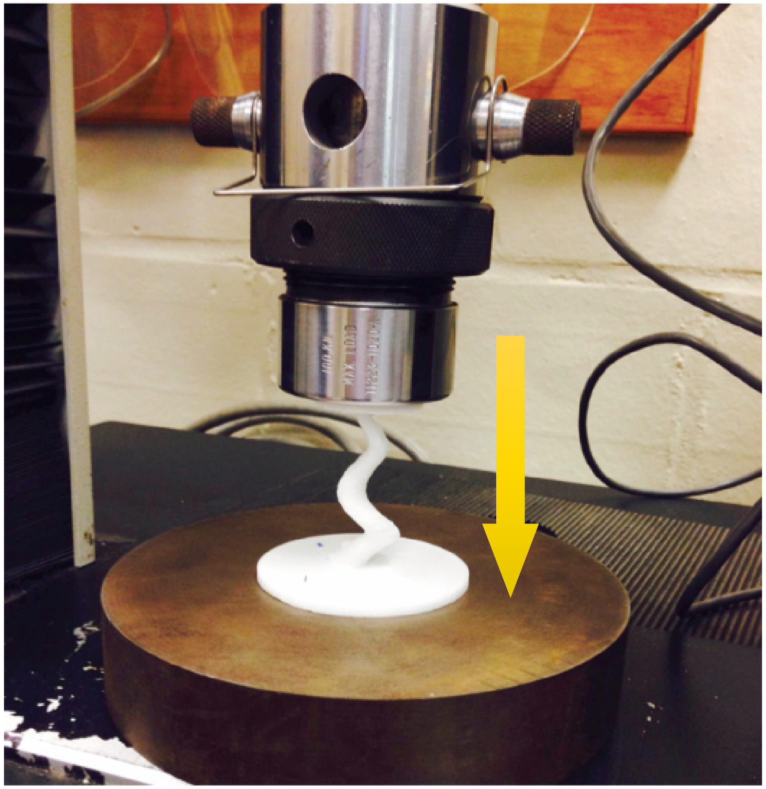
Gallery of Results – 200% More Force


Physical Validation

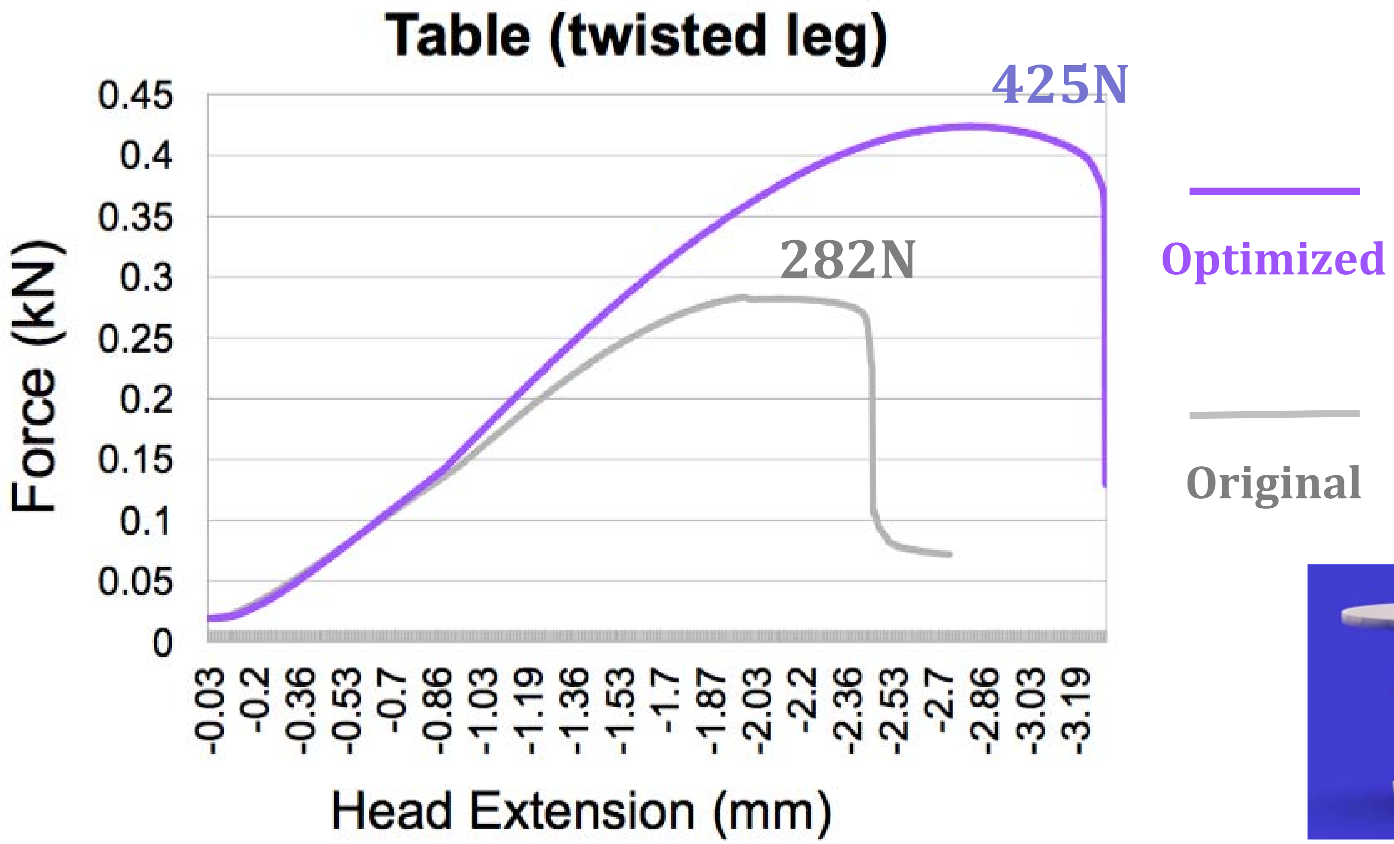




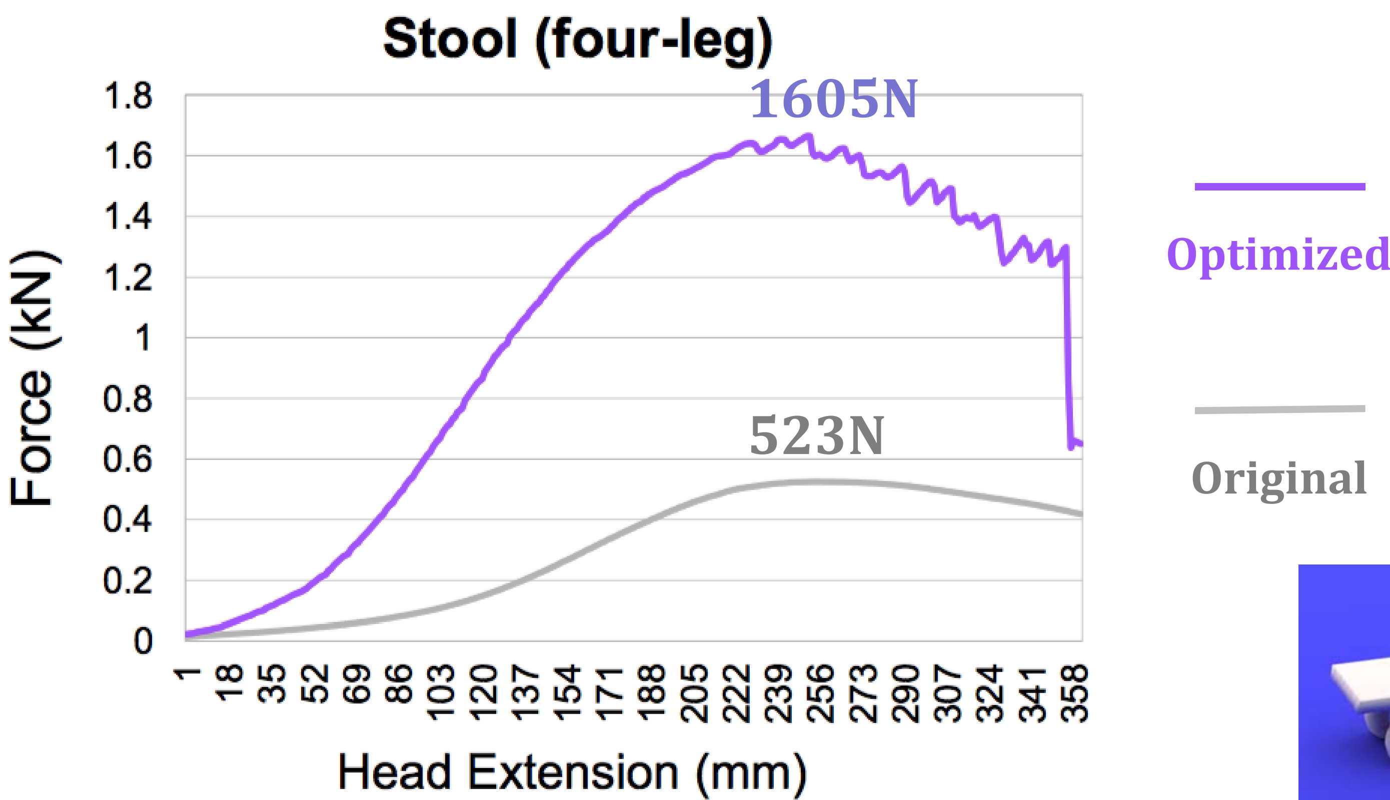
Physical Validation

- Optimized shapes withstand 100% more force.
- PLA material, 100% infill, 100 micron resolution
- Equalize volume for fair comparisons.

Physical Validation



Physical Validation



Conclusion

- An algorithm to directly optimize a 3D mesh to make it withstand specified external force.
- Integrates optimization and physics simulation in a unified framework.
- Derivations of analytic gradient and Hessian of the objective function.
- Applications to printable object design.

Limitations and Future Work

- Performance, convergence speed
- Tetrahedralization quality
- Incorporating higher-order Laplacians [BS08]
- Applications to other design goals, such as improving aerodynamic properties of shapes.

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Direct Shape Optimization for
Strengthening 3D Printable Objects